Below Resonance Operation of the Strip Line Y-Circulator

SUMMARY

Below resonance operation of a strip line Y circulator is analyzed and the limitation in choice of the internal field and of the magnetization $4\pi M_s$ is pointed out. Bosma's[1] design equations are reviewed, and a change in design procedure is suggested.

In a strip line circulator the two ferrite discs are subject to a magnetic biasing field H_i. The circulating properties can be obtained with $H_i > H_o$ and with $H_i < H_o$ where H_o denotes the magnetic field at which the ferromagnetic resonance occurs at the signal frequency ω. This resonance field is defined by the equation $\gamma H_o = \omega$ where $\gamma = 2\pi \times 2.8 \times 10^6$ rad/sec. oersted.

If $H_i < H_o$, the circulator is said to perform below resonance. In this case, however, the magnitude of H_i is bounded within rather narrow limits: H_i must be large enough to completely saturate the ferrite disks in order to avoid low field losses. Thus H_i must be larger than the anisotropy field: $H_i > H_a$. On the other hand, the effective permeability,

$$\mu_{\text{eff}} = \frac{\mu^2 - k^2}{\mu}$$

which depends on H_i, must be positive. Immediately below resonance, μ_{eff} is negative, and a cut-off of transmission occurs due to the imaginary intrinsic impedance $\sqrt{\mu_{\text{eff}}/\epsilon}$. This imposes an upper limit on H_i.

The diagonal and off-diagonal components of the permeability tensor, neglecting losses, are

$$\mu = 1 + \frac{H_i 4\pi M_s}{H_i^2 - H_o^2} \tag{1}$$

$$k = \frac{H_o 4\pi M_s}{H_i^2 - H_o^2}. \tag{2}$$

Below resonance $H_i < H_o$. From (1) and (2) the relative effective permeability μ_{eff} becomes

$$\mu_{\text{eff}} = \frac{\mu^2 - k^2}{\mu} = \frac{(H_i + 4\pi M_s)^2 - H_o^2}{H_i^2 + H_i 4\pi M_s - H_o^2}. \tag{3}$$

From this it follows that for $H_i = H_o - 4\pi M_s$, $\mu_{\text{eff}} = 0$. For $H_i > H_o - 4\pi M_s$, μ_{eff} is negative. For operation below resonance, therefore, H_i must be chosen between the following limits:

$$H_a < H_i < H_o - 4\pi M_s. \tag{4}$$

This relation also limits the magnitude $4\pi M_s$. As for the magnitude of μ_{eff} corresponding to the magnetic field limited by (4), it is easy to see that μ_{eff} is always smaller than unity. This, together with the fact that for ferrites ϵ_f is of the order of 12 to 15, proves that the intrinsic impedance of ferrites operating below resonance is very small: $\zeta_{\text{eff}} = \sqrt{\mu_0 \mu_{\text{eff}}/\epsilon_0 \epsilon_f} < \sqrt{\mu_0/\epsilon_0}$. Eq. (4) has been found for the lossless case. The limitations on real ferrites are more stringent,

$$H_a < H_i < H_o - 4\pi M_s - 2\Delta H \tag{5}$$

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[1] H. Bosma, "On stripline Y-circulation at UHF," IEEE Trans. on Microwave Theory and Techniques, vol. MTT-12, pp. 61–72; January, 1964.

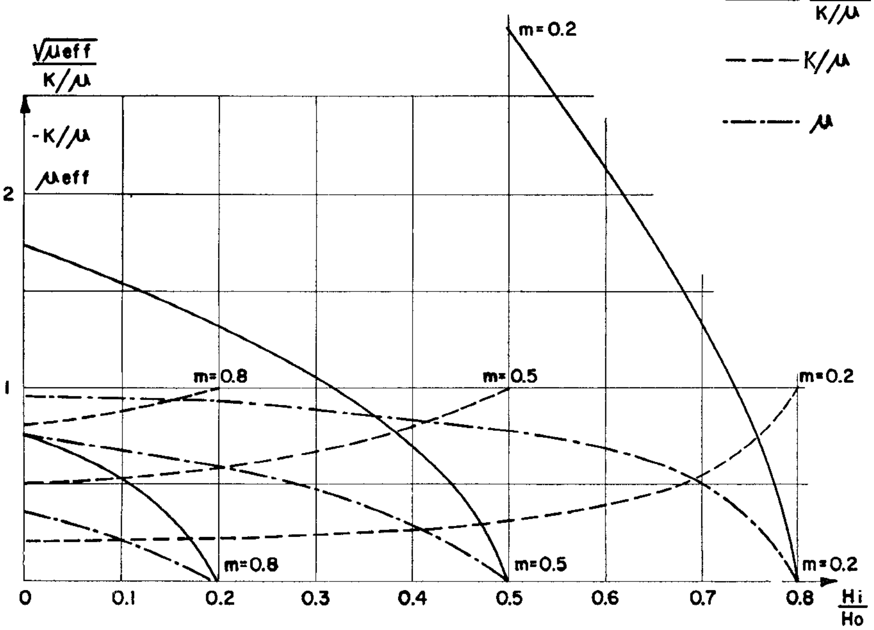

Fig. 1—Dependence of μ_{eff}, k/μ, and $\sqrt{\mu_{\text{eff}}}/k/\mu$ on the normalized biasing field H_i/H_o. In the figure $m = 4\pi M_s/H_o$.

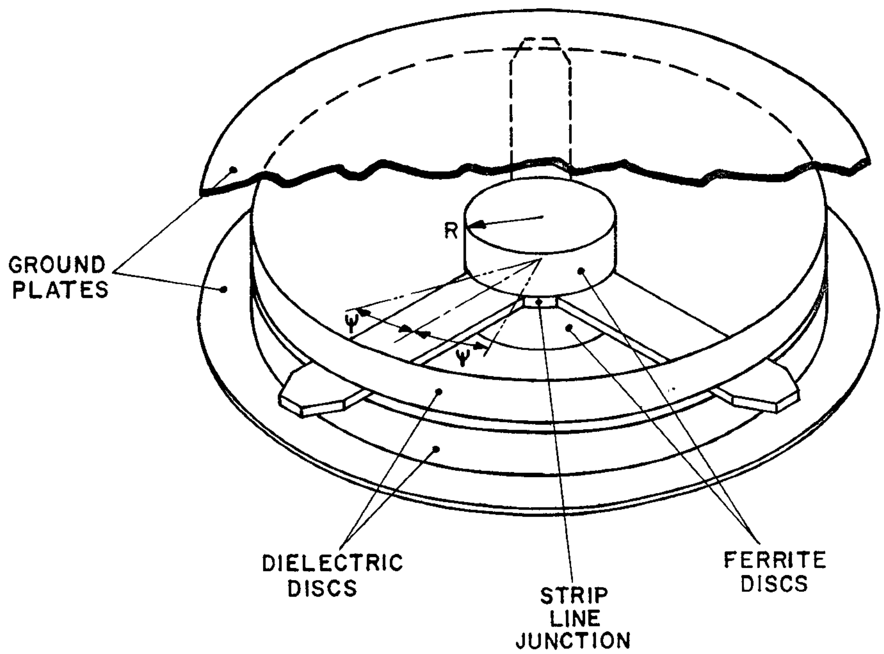

Fig. 2—Y Strip line circulator and meaning of the angle ψ.

where ΔH is the line width of the ferrite. In fact, if the losses are not neglected, the expression for μ_{eff} becomes:[2]

$$\mu_{\text{eff}} = \frac{(H_i + 4\pi M_s)^2 - H_o^2 + 2j\Delta H(H_i + 4\pi M_s)}{H_i(H_i + 4\pi M_s) - H_o^2 + 2j\Delta H(H_i + 2\pi M_s)}. \tag{6}$$

From (6) it can be seen that in the neighborhood of a certain H_r which almost equals to $H_o - 4\pi M_s$ the effective permeability μ_{eff} has a resonance. The width of this resonance can be determined approximately by setting the real and imaginary parts of the numerator equal. The increment of H_i field needed to change μ_{eff} from its resonant value ($\mu_{\text{real}} = 0$) to the point where $\mu_{\text{real}} = \mu_{\text{imag}}$ is approximately equal to the line width ΔH.

When H_i approaches the resonant magnitude $H_i = H_o - 4\pi M_s$, the losses in the ferrite discs increase. In order to keep them low, the performance of the circulator must be limited to the magnitudes of H_i reasonably smaller than $H_0 - 4\pi M_s$. Rather arbitrarily, this safety factor has been chosen as $2\Delta H$ in (5). Keeping the above defined limitations of H_i and μ_{eff} in mind, it becomes evident that one of Bosma's[1] de-

[2] A. G. Gurevich, "Ferrites at Microwave Frequencies," Consultants Bureau, New York, N. Y., p. 169; 1963.

sign equations,

$$\frac{\sqrt{3}\,1.84}{\pi}\,\frac{\psi}{\sqrt{\epsilon_f}}\,\frac{\sqrt{\mu_{\text{eff}}}}{k/\mu} = 1, \tag{7}$$

is not easily satisfied when operating below resonance in L or S band. In fact, since $\sqrt{3}\cdot 1.84/\pi \approx 1$ and $\psi \leq 1$ which is the angle defined by Fig. 2, the quantity

$$\frac{1}{\sqrt{\epsilon_f}}\,\frac{\sqrt{\mu_{\text{eff}}}}{k/\mu}$$

must equal or exceed unity to satisfy (7). At low frequencies, where $4\pi M_s/H_0$ cannot be much smaller than one, this may be impossible. See Fig. 1. At higher frequencies it is inconvenient to make the quantity $(1/\sqrt{\epsilon_f})(\sqrt{\mu_{\text{eff}}}/k/\mu)$ approximate unity because this would require a rather small K/μ. Small values of K/μ reduce the bandwidth of the circulator. The graph in Fig. 1 shows how μ_{eff}, k/μ, and $\sqrt{\mu_{\text{eff}}}/k/\mu$ vary with H_i/H_o for three particular values of $4\pi M_s/H_o$. Thus, instead of using the two design equations of Bosma,[1] (7), and

$$\frac{R}{\lambda} = \frac{1.84}{2\pi\sqrt{\mu_{\text{eff}}\epsilon_f}} \tag{8}$$

and solving them to find H_i and the radius of the ferrite disc R, the design can be made in the following way:

1) Choose H_i in accordance to (5).
2) Calculate μ_{eff} from (3).
3) Find R from (8).
4) Match the ports of the circulator to the 50 ohm external line.

The impedance-transforming arrangement should have as wide a frequency band as possible so as not to limit the bandwidth of the circulator. The use of a half wave tapered line transformer and dielectric discs (Fig. 2) of high dielectric constant ϵ_d gives a rather compact solution. In this case ϵ_d must satisfy the relation

$$\sqrt{\frac{\mu_0}{\epsilon_0}}\,\frac{1}{\sqrt{\epsilon_d}} = \sqrt{\frac{\mu_0}{\epsilon_0}}\,\frac{\sqrt{3}\,1.84}{\pi}\,\psi\,\sqrt{\frac{\mu_{\text{eff}}}{\epsilon_f}}\Big/\frac{k}{\mu} \tag{9}$$

where $\sqrt{\mu_0/\epsilon_0} = 120\pi =$ the intrinsic impedance of free space. The use of $\lambda/4$ transformers offers another solution. In this case the dielectric constant of the dielectric disks must satisfy the relation

$$\left[\sqrt{\frac{\mu_0}{\epsilon_0}}\,\frac{\sqrt{3}\,1.84}{\pi}\,\psi\,\sqrt{\frac{\mu_{\text{eff}}}{\epsilon_f}}\Big/\frac{k}{\mu}\right]\sqrt{\frac{\mu_0}{\epsilon_0}} = \left(\sqrt{\frac{\mu_0}{\epsilon_0}}\,\frac{1}{\sqrt{\epsilon_d}}\right)^2. \tag{10}$$

The width of the strip line and the distance between the ground plates must be such that the characteristic impedance of the strip line is 50 ohms when air insulated.

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Surface Wave on a Perfectly Conducting Plane Covered with Magnetoplasma

It has been found by Ishimaru[1] and Seshadri[2] that a unidirectional TEM surface wave is trapped along a perfectly conducting plane covered with *transversely* magnetized semi-infinite plasma.

This communication gives a brief description of a surface wave which is found to be trapped along a perfectly conducting plane covered with *longitudinally* magnetized semi-infinite plasma. When the plasma is magnetized in the direction of propagation, the electromagnetic fields can not be separated into TE or TM modes, and are given by a combination of two characteristic wave modes having each different, but interrelated, characteristic values to satisfy Maxwell's equations for magnetoplasma. The combination is determined by the boundary conditions.

The coordinate is chosen so that the perfectly conducting plane is located in the y-z plane ($x=0$) and the z axis is the direction of propagating waves or the direction of magnetization.

All the field components are assumed to be independent of the y coordinate and have the time and z dependence of $e^{j\omega t - jk_0hz}$ (h: relative axial propagation constant). The electromagnetic fields satisfying Maxwell's equations and the boundary conditions can be expressed by a combination of a right-handed plane wave mode and a left-handed plane wave mode with the same axial propagation constant h. The resulting field equations are given by

$$\left.\begin{aligned}
E_x &= Z_0(s_1^2+\epsilon_3)\{(s_1^2-h^2+\epsilon_1)e^{-s_1k_0x} \\
&\qquad -(s_2^2-h^2+\epsilon_1)e^{-s_2k_0x}\} \\
E_y &= -j\epsilon_2Z_0(s_1^2+\epsilon_3)\{e^{-s_1k_0x}-e^{-s_2k_0x}\} \\
E_z &= jZ_0s_1h(s_1^2-h^2+\epsilon_1)\{e^{-s_1k_0x}-e^{-s_2k_0x}\} \\
H_x &= j\epsilon_2h(s_1^2+\epsilon_3)\{e^{-s_1k_0x}-e^{-s_2k_0x}\} \\
H_y &= \epsilon_3h(s_1^2-h^2+\epsilon_1) \\
&\qquad \cdot\left\{e^{-s_1k_0x}-\frac{s_1}{s_2}e^{-s_2k_0x}\right\} \\
H_z &= -\epsilon_2(s_1^2+\epsilon_3)\{s_1e^{-s_1k_0x}-s_2e^{-s_2k_0x}\}
\end{aligned}\right\}, \tag{1}$$

with the dispersion equation

$$s_1^2+s_2^2+s_1s_2-h^2+\frac{\epsilon_1^2-\epsilon_2^2}{\epsilon_1}=0, \tag{2}$$

where s_1 and s_2 are relative transverse propagation constants normalized with respect to free space propagation constant k_0 and are solutions of the quadratic equation:

$$s^4 - s^2\left[h^2\left(1+\frac{\epsilon_3}{\epsilon_1}\right)+\frac{\epsilon_2^2}{\epsilon_1}-\epsilon_1-\epsilon_3\right] + \frac{\epsilon}{\epsilon_1}\left[(h^2-\epsilon_1)^2-\epsilon_2^2\right]=0. \tag{3}$$

Manuscript received June 3, 1964.

[1] A. Ishimaru, "The effect of a unidirectional surface wave along a perfectly conducting plane on the radiation from a plasma sheath," presented at the 2nd Symp. on the Plasma Sheath, Boston, Mass.; April, 1962.

[2] S. R. Seshadri, "Excitation of surface waves on a perfectly conducting screen covered with anisotropic plasma," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-10, pp. 573–578; November, 1962.

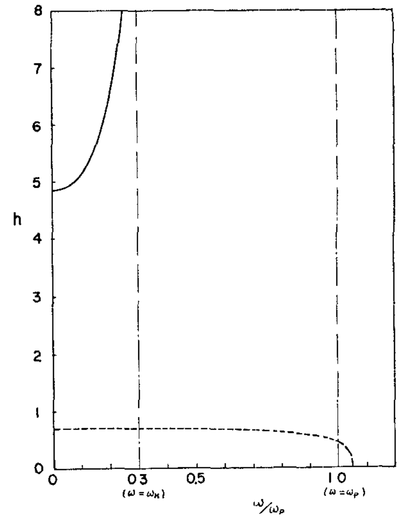

Fig. 1—Frequency characteristics of the relative propagation constants for $\omega_H < \omega_p$, —— : trapped wave; — — : plane wave; - - - : improper wave.

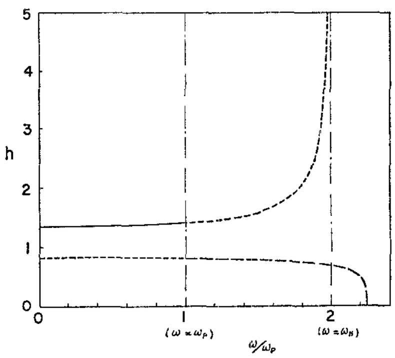

Fig. 2—Frequency characteristics of the relative propagation constants for $\omega_H > \omega_p$, —— : trapped wave; — — : plane wave; - - - : improper wave

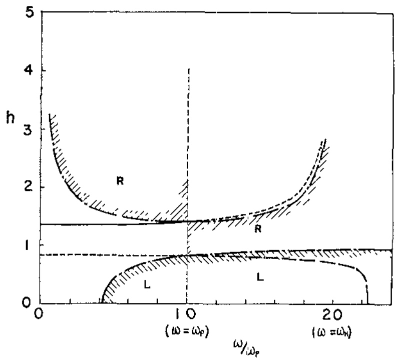

Fig. 3—Illustrating the relation between the propagation constant h of the trapped wave and the permissive axial propagation constants of two characteristic plane waves in a free magnetoplasma. Symbols R and L denote the regions of the permissive axial propagation constants of a nonattenuating right-handed and a left-handed plane wave, respectively.